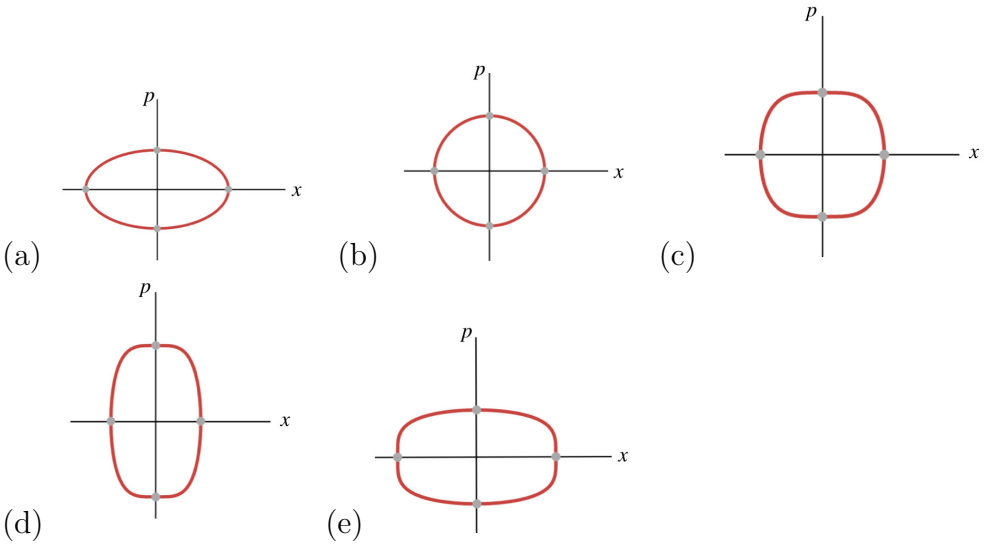
1. Three people (nicknamed A, B, C) are repeatedly tossing a coin in turns, starting with A, in that order (A-B-C-A-B... *etc.*). The first one to get a heads will win. The coin is biased and has 20% chance of falling heads at each toss. Find the probability that A wins the game.

2. In a laboratory frame a particle ($A$) of rest mass $m_A = 4m$ moving at a speed $v_A = \frac{3c}{5}$ along $+x$ axis collides heads on with another particle ($B$) of rest mass $m_B = 3m$ moving at a speed $v_B = \frac{4c}{5}$ along $-x$ axis. If a single particle ($C$) is produced in the collision find the rest mass of the particle $C$ (ignoring any energy loss due to the collision).

3. Consider a particle, moving in one dimension under the influence of the potential $V(x) = \frac{1}{4}kx^4$. What will the phase trajectory of the particle look like, if the numerical value of $k$ is $2/m$? A few example trajectories are given below:




4. The eigenvalues $\lambda_m$, $(m = 0, 1, 2, \cdots, n-1)$ of the following $n \times n$ matrix are

$$\begin{pmatrix} a_1 & a_2 & \cdots & a_n \\ a_n & a_1 & \cdots & a_{n-1} \\ \cdot & \cdot & & \\ \cdot & & & \\ a_2 & a_3 & \cdots & a_1 \end{pmatrix}$$

5. Consider a three-state system having energies equal to 0, $k_BT$, $3k_BT$, with $k_B$ being the Boltzmann constant and $T$ being the temperature. If $N$ classical particles are distributed among the three states, and the average energy of the system is $200k_BT$, find the approximate value of $N$.

6. A quantum harmonic oscillator with frequency $\omega$ is in a state represented by wave function $\psi(x) = \phi_1(x) - 2\phi_2(x) + 3\phi_3(x)$, where $\phi_n$ represents eigenfunction of the harmonic oscillator corresponding to $n^{\text{th}}$ excited state. Find the expectation value of energy in the state $\psi(x)$ in units of $\hbar\omega$.

7. The potential energy of a diatomic molecule is given in terms of the interatomic distance $r$ by the expression, $U(r) = -a/r^2 + b/r^{10}$. Given: $a = 1.44 \times 10^{-39}$ J-m$^2$ and $b = 2.19 \times 10^{-115}$ J-m$^{10}$, find the equilibrium spacing of the two atoms.

8. The internal energy of a gaseous system is given by

$$U = 2.5PV + K$$

where $P$ and $V$ denote pressure and volume of the gas and $K$ is a constant. What is the equation for the path of system in $P - V$ plane, when the system is undergoing an adiabatic change? (*Hint*: find an equation of the form $P^aV^b =$ constant where you have to identify $a$ and $b$).