Science Academies’ Refresher Course on Statistical Physics

Dates: *9 – 23 December, 2018*
Venue: *Ramakrishna Mission Vivekananda University, Belur Math, India*

Academic Report

The refresher course was held in the Department of Physics, Ramakrishna Mission Vivekananda University. For selecting participants for the course, all of 110 applications received within the application deadline were considered. In the end, 29 (local: 3, outstation: 26) selected participants attended the course.

In the course, Professor Mukunda gave a set of eight lectures + tutorials on the subject of classical mechanics. Topics covered include: Physical basis of Galilean-Newtonian Mechanics and Newton's Laws of Motion, discussion of the concept of inertial frames, transition from Newtonian to Lagrangian form of mechanics, roles of constraints and generalized coordinates, number of degrees of freedom, Hamilton's Principle of stationary action both without and with endpoint variations, point transformations as covariance group of Lagrangian mechanics, passage from Lagrangian to Hamiltonian mechanics, concept of phase space for a given configuration space, Poisson brackets, canonical transformations as covariance group of Hamiltonian mechanics, descriptions of infinitesimal and finite canonical transformations, generating functions and Caratheodory's theorem, properties of action functional as a function, the connections between continuous symmetries, constants of motion and corresponding canonical transformations, a brief discussion of Hamilton-Jacobi theory, and the motivations underlying the Dirac theory of constrained systems.

Professor Mustansir Barma gave a set of six lectures on the subject of stochastic dynamics, supplemented by two tutorials. The elements of probability theory were reviewed, including probability distributions, sums of random variables, the law of large numbers and the central limit theorem. Properties of the random walk were derived combinatorially and, in the continuum limit, through an equation of motion for the probability density, resulting in the diffusion-drift equation. The question of returns to the origin was considered, and the distribution of first return times was computed. Turning to stochastic dynamics, the Langevin equation for the velocity of a Brownian particle was written down and solved, and shown to reduce to the Maxwell-Boltzmann distribution at long times. Finally, the passage to a Fokker-Planck equation was demonstrated for Brownian dynamics as well as more general processes. Results for the Ornstein-Uhlenbeck process as well as for correlated noise were given.

Professor Soumitro Banerjee gave four lectures on dynamical systems. The first lecture was devoted to introducing discrete and continuous-time dynamical systems, with examples taken from the field of ecology. Other topics covered include: Two and three-dimensional flows, phase portrait, Lorenz system, limit cycles, chaos. In lecture 2, several animation of chaotic dynamics was shown, followed by an explanation of concepts such as Poincare section, one-dimensional maps and fixed-point analysis, bifurcation diagram of maps, types of bifurcation in maps. The lecture ended with a discussion of examples of maps. Lecture 3 was devoted to discussions on Henon map and period-doubling bifurcation. Topics covered in lecture 4 included representation of maps in matrix form, fractals, contraction mapping and examples.

Professor P K Mohanty gave a series of eight lectures + tutorials on the topic of nonequilibrium statistical mechanics. Starting with a bird’s eye view of the subject, the concept of matrix product steady states was introduced using the Ising model as an example, followed by discussions of the transfer matrix method, crossing of eigenvalues, phase transitions and their non-occurrence in one dimensional systems with short-range interactions, non-equilibrium dynamics, the Master equation and steady states. The zero-range process was introduced as an exactly solvable model, and the possibility of condensation transition in the model was discussed in detail. Subsequently, the totally asymmetric exclusion process was introduced to demonstrate the beauty of matrix product steady states. Explicit derivation of the matrix algebra was also discussed.

Dr. Shamik Gupta gave three lectures on dynamical systems and six lectures on equilibrium statistical mechanics, in addition to conducting three tutorial sessions. The principles of dynamical system theory and thermodynamics were discussed in detail and the need to go for a statistical description was clarified. Lecture 1 was devoted to introducing the framework of dynamical systems with examples and the concept of fixed points and bifurcation. In lecture 2, one-dimensional flows and the normal forms of bifurcation in one dimension were discussed. Lectures 3 and 4 were aimed as an exposition of conservative dynamical systems, with explanation of concepts such as integrability and the Liouville-Arnold theorem, the

Kolmogorov-Arnold-Moser theorem, ergodicity, mixing, and chaos. Lecture 5 was a recapitulation of the basic concepts of thermodynamics. In lectures 6–8, the foundation of classical equilibrium statistical mechanics and, in particular, elements of the ensemble theory and the three ensembles, the microcanonical, the canonical, and the grandcanonical ensembles, were discussed. In these lectures, it was pointed out how statistical mechanics bridges the gap between microscopic dynamics and macroscopic observables.

In his 8 lectures + tutorials, Dr. Arnab Sen discussed aspects of quantum statistical mechanics and phase transitions using examples. In the first lecture, the basic postulates of quantum statistical mechanics were introduced. The density matrix of one particle in a box and two non-interacting identical (both fermions and bosons) particles in a box was then computed. The classical limit of these computations and the quantum effects induced at finite temperature were pointed out. In the second lecture, treatment of a single quantum harmonic oscillator was introduced. The interpretation of the quantum harmonic oscillator as a bosonic problem using ladder operators was discussed. The normal modes for monoatomic and diatomic crystals were also worked out. In the third lecture, the specific heat of an insulator was calculated using the model of free phonons and the universality of the T^3 behaviour for insulators at low temperatures was stressed. The various approximations in the Einstein and the Debye model were pointed out. In the fourth lecture, the power distribution and specific heat of a gas of photons at equilibrium (blackbody radiation) was calculated. The number fluctuations of the photon gas was also computed. The phenomenon of Bose-Einstein condensation of a system of free bosons was shown and the BEC temperature computed. In the fifth and sixth lectures, various concepts regarding phase transitions and order parameters were discussed using the examples of one dimensional, two dimensional and the infinite-dimensional Ising model, respectively. The absence of a finite temperature phase transition was demonstrated for the one dimensional Ising model using a free energy minimization argument. The Peierls argument was given for the two dimensional Ising model. The high temperature and low temperature duality of the two dimensional Ising model was shown using the two expansion for the partition function and the exact critical temperature was then computed. Mean field theory was then introduced and the various approximations were pointed out. The concept of universality in phase transitions was briefly discussed. In the two tutorials, various doubts regarding the lectures were discussed and the participants were encouraged to solve problems illustrating certain concepts.